**Free Energy with B?**

**Landau Free Energy**

So I’d like to calculate the free energy via a diagrammatic expansion. But I’m going to do it with the path integral approach. Not sure this is going to work. So let’s start with 2nd quantized Hamiltonian in position space, with magnetic field.



Referring to the Stat Mech/G, Z as Path Integral file, and QM/Time-Dependent/Many Identical Particles/Propagator as Path Integral file, we have:



where,



and K = H – μN. Filling in our particular H, we get:



Going to have to perturbatively evaluate the contributions from A, so let’s just make the perturbative expansion here. I’m going to define some stuff. First:



and,



Then can say,



I’ll need to separate out the A’s. So let’s look at:



Note the ∇ acts on everything to its right, so that first ∇ would act on A, and on anything that comes after. So now we have:



Now we’re in a position to calculate Ξ perturbatively. We will separate out the free part of S from the interacting part of S (the one with the A’s):



So this is the general perturbative expansion of Ξ, which we can do with Feynman diagrams, symmetry factors, and all that stuff. Might also recall the general formula,



where <φn> is the nth moment of φ, and <φ>n is the nth cumulant of φ. So we can say,



Now low let’s look at the Feynman rules,

**Feynman rules**

Okay well now we need the Feynman rules. To that end I’m going to simplify the action a little. Remember that first grad acts on everything to its right…but we can IBP to put the derivative on the .



where,



And then the Feynman rules are:

Diagram

Description automatically generated

and the GF is given by:



and the gradients on that (1) vertex would act on the x of the GF that it’s pointing to. So the left one acts on the 2nd argument of GF that connects to the left, and the one on the right acts on the 1st argument of the GF that connects to the right. And we’d add the two gradients. So it’s really an operator. It’s better, though, to convert to Fourier space rules. See the FT file for ideas, and also might check out the Cond Mat / Metals / Electrons and Impurities / Conductivity / Quantum file for another instance where the leg gets a momentum factor.

Diagram, schematic

Description automatically generated

Note the A2(q,iνm) is to be interpreted as the spatio-temperal FT of A2(x,τ) – not obvious from the notation, granted. The frequency index on the A’s is bosonic, because it will be the difference between an incoming and outgoing fermion leg, and the difference between two fermion/odd frequencies is a bosonic/even frequency. And for our case, we’ll say A is actually time-independent. Also, for all of these rules, there will be a net factor of 2 for every closed loop, since that will imply a Trace over the GF matrix, which is diagonal and so will just produce two copies of the same thing. As usual we conserve energy/frequency and momentum/wavenumber at every vertex, and sum over all frequencies and wavenumbers, with additional factor of 1/β and 1/V for each k and iωn d.o.f. Oh and (-1) for every Fermion loop. Anyway, so we’ll recall our Free Energy is given by:



and up to second order in A, this will correspond to the following diagrammatic expansion,

Diagram

Description automatically generated with medium confidence

So let’s get started…

**O(A0) term**

First we have to get Ξfree. This is (see Path Integral files):



We’ll recall **G**0-1 is given by,



So,



And the eigenvectors of G11|0-1 are given by:



where n1/2 are half-integers, and this is because Fermions are anti-periodic over boundary (0, β). BTW, there is no |τ>, of course, in a QM Hilbert Space, but we can use that notation here in our purely mathematical HS, as our operators in the exponent of our integral act on both space and time. In this basis, the matrix reads:



where ξk = k2/2m – μ. Since in this basis, the matrix is diagonal, the ln() can go through, and we have:



To evaluate the frequency sum, we’ll employ the method we used in the Stat Mech folder/GF,Z file. We use the identity (see Stat Mech/Math Appendix),



and ignore the convergence issues at |z| → ∞. The ln(z) function has no poles, and we’ll have to integrate around the branch cut Re(z) > ξk. Contour looks like this, but infinitely large.

Chart, diagram

Description automatically generated with medium confidence

Then we have, using the principle value of the ln(z) function [ln(x+i0+) = ln|x|, and ln(x – i0+) = ln|x| + 2πi0+]:



And so altogether,



So then up till now, our L is:



which is indeed the correct result (the 2 accounts for two orientations of spin).

**O(A1) term**

In order to get the magnetization, we need to expand this out to at least 2nd order in A. There are three diagrams that make contributions at this level. The first is,

Diagram

Description automatically generated

we get:



Since the summand is odd.

**O(A2) terms**

The next A-term is:

Diagram

Description automatically generated

This translates to:



Using the frequency sum formula,



So this comes to:



And we have:



And last,

Diagram, text

Description automatically generated

which is:



Well,



So,



Then using the identity,



we can do the frequency sum,



Should be able to ignore the δq=0 term in favor of the other, especially since the other reduces to it in the limit q = 0, so we’ll just say,



We can simplify a little more. Let’s change variables k → k + q/2.



Well I’ll stop here,



Well, this is not something I want to evaluate. Well, say A(q) is strongly damped about q = 0. Then we can probably get away with:



Now we’ll recognize the (1/V)Σq A(q)A(-q) as just the integral of A(x)2 over all space. So now we have:



And from the Excitations folder/Properties, we know that in 3D we have:



So we can say,



So yeah. But I suspect that we really need to go out to higher order in q to get a decent result.

**Final Result**

But altogether, then our partition function would be:



Not going to try to get M from this. But if want L, then can do, out to 2nd order,



So then,



and if we use that approximation for Ξ2b, then we could write this as:



But this won’t suffice to get us the correct magnetization. So we do need to go out to higher order in q for Ξ2b. But not going to do that, because I don’t feel like it.